SIPL SIGNAL & IMAGE PROCESSING LAB

Technion Israel Institute of Technology

Multipath Medium Identification Using Efficient Sampling Schemes

Student: Rami Cohen

Supervisor: Kfir Gedalyahu

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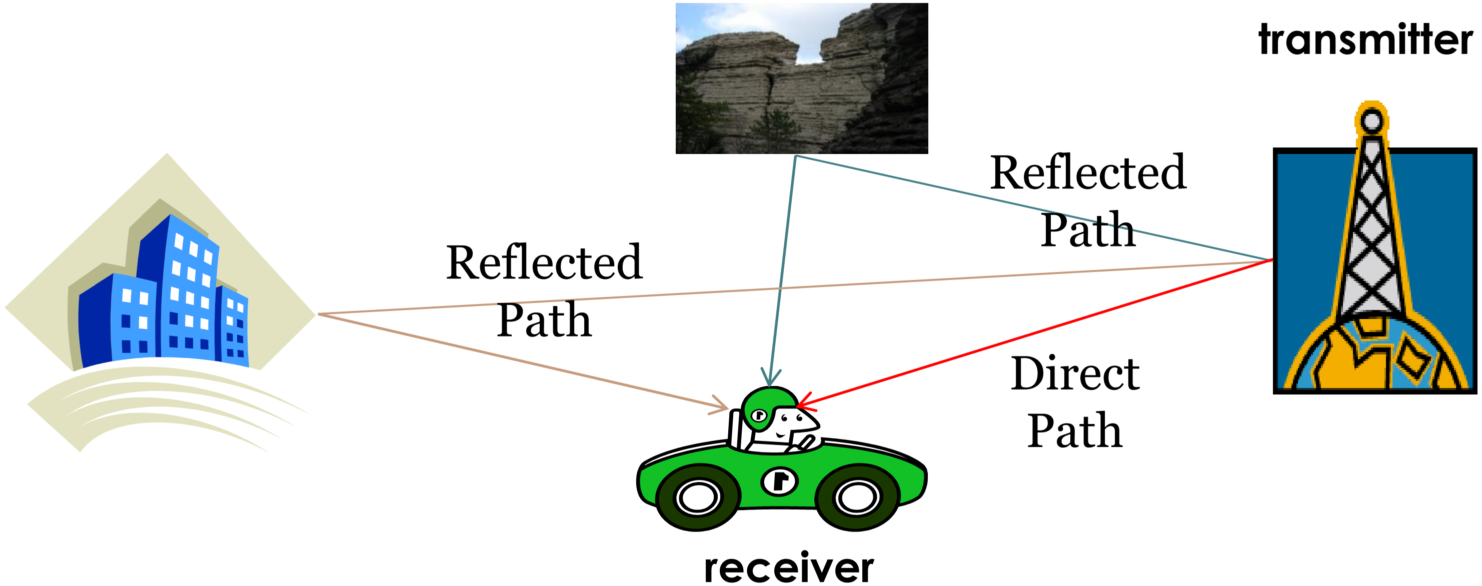
Project Overview



- In this work, a new method for time delay estimation from samples taken at sub-Nyquist rate has been investigated

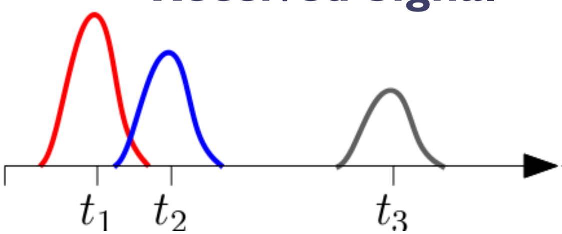
The Model

Transmitted Signal

$$x_T(t) = \sum_{n \in \mathbb{Z}} g(t - nT)$$

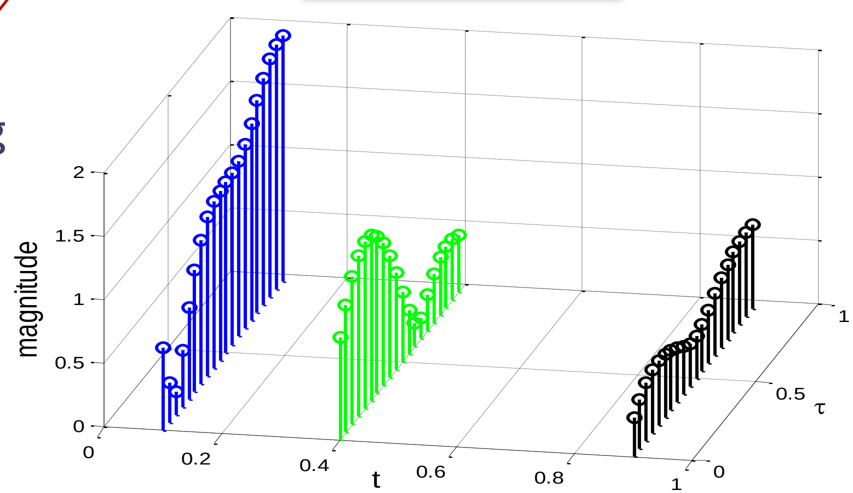
Multipath Medium

Received Signal



$$x_R(t) = \sum_{k=1}^{K} \sum_{n \in \mathbb{Z}} a_k[n]\, g(t - t_k - nT)$$

Probing Rate



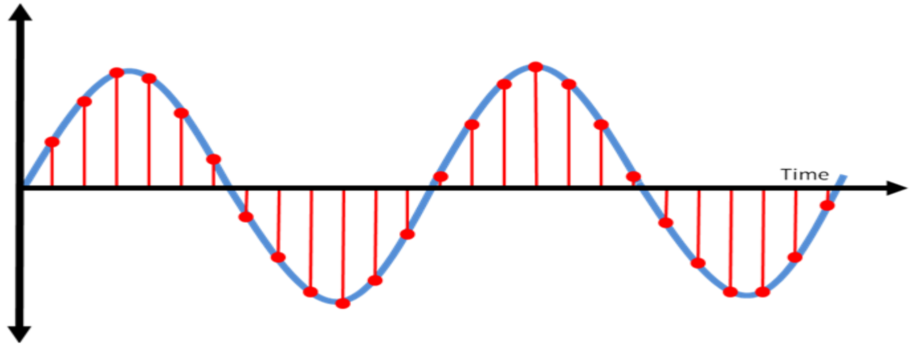
kth path's time-variant gain coefficients

kth path's propagation delay

Goals

$$x_R(t)=\sum_{k=1}^{K}\sum_{n\in\mathbb{Z}} a_k[n]\,g(t-t_k-nT)$$

- Recovering medium parameters:
  - Delays - t_k
  - Gain coefficients - $a_k[n]$
- Using low sampling rate (samples of $x_R(t)$)
- Practical implementation

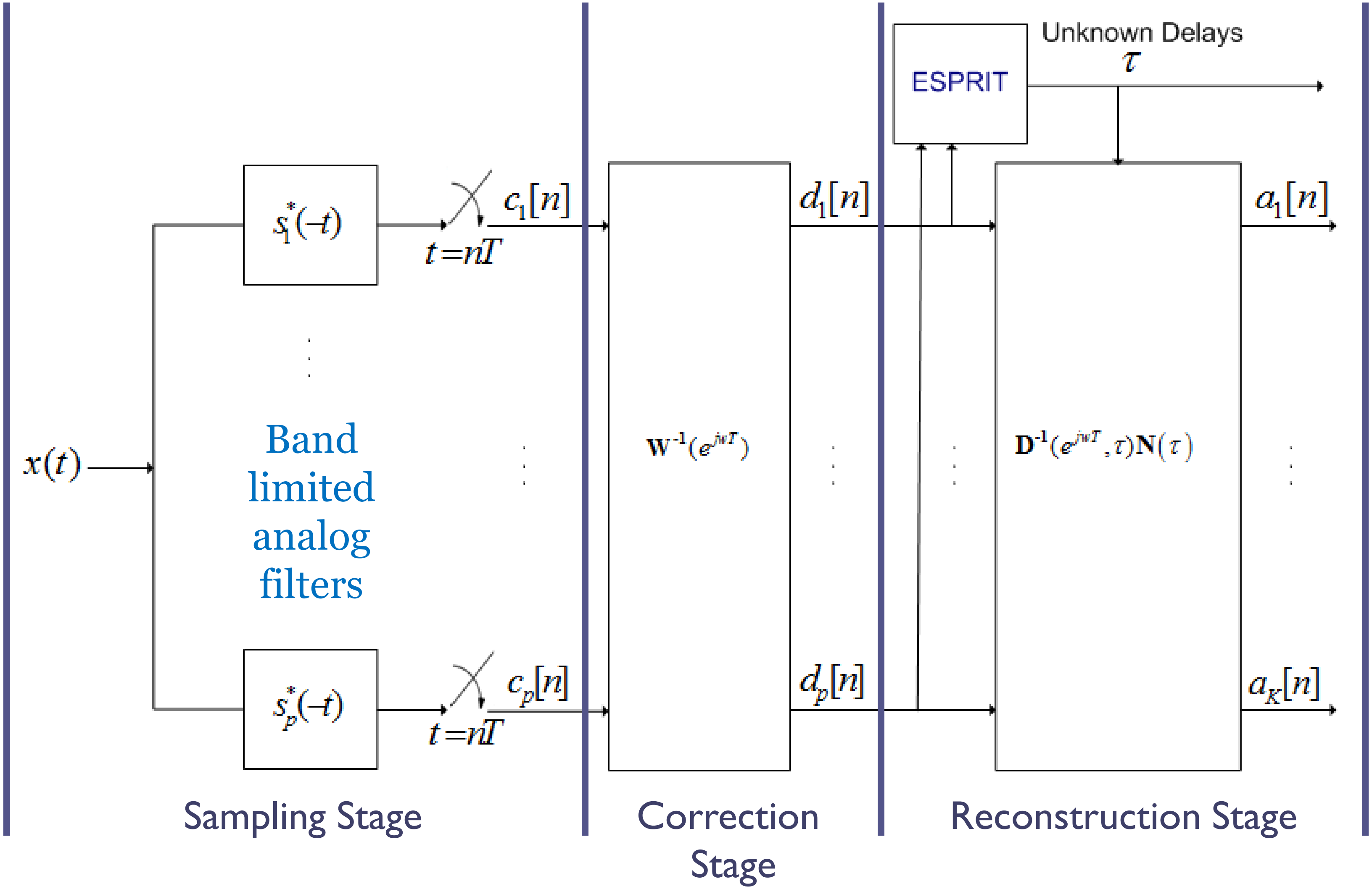


Sampling Rate Reduction

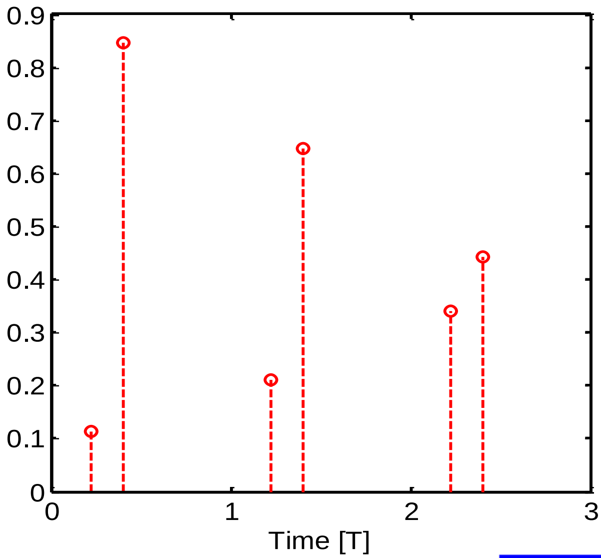
- Example: characterizing UWB channel
  - Pulse bandwidth: $B = 1GHz$
  - Probing rate: $1/T = 2MHz$

- Assuming $K = 10$, we can get a sampling rate of $2K/T = 40MHz$, which is only 2% of Nyquist-rate
- Sampling rate reduction:
  - Lower computational load, more precise ADC
  - lower power consumption

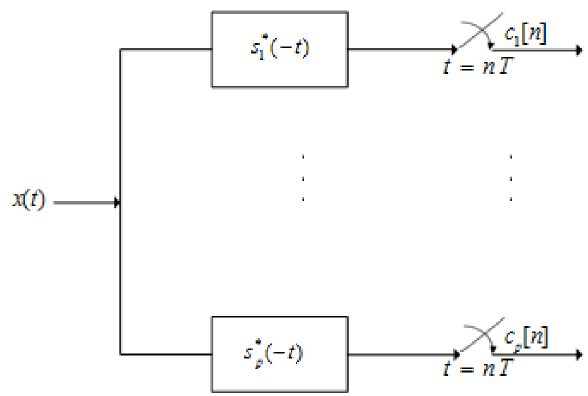
Proposed Algorithm

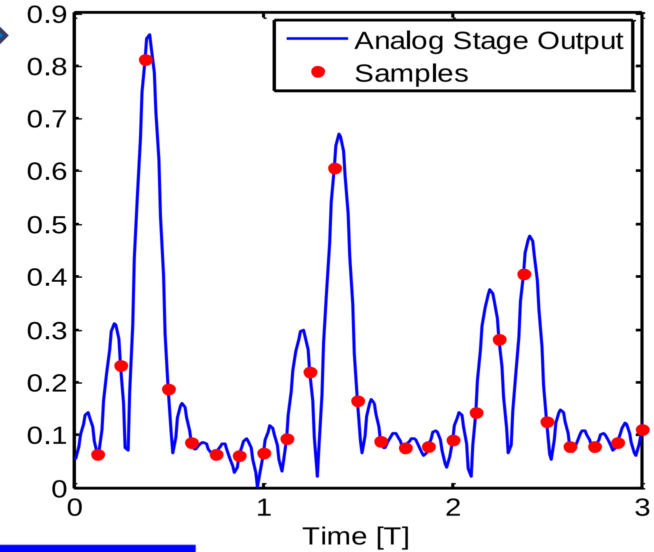


Sampling Stage



Smoothing and sampling





$$\mathbf{c}\left(e^{j\omega T}\right) = \mathbf{M}_{SG}\left(e^{j\omega T}, \tau\right)\mathbf{a}\left(e^{j\omega T}\right)$$

DTFT of sampling sequences

Depends on the unknown delays $\tau = \{t_1, t_2, \ldots, t_K\}$

DTFT of $a_k[n]$

- Strategy:
  - Recovering the unknown delays from the sampling sequences
  - Recovering the gain coefficients using a digital correction filter

Reconstruction Stage

$$\mathbf{b}\left(e^{j\omega T}\right) = \mathbf{D}\left(e^{j\omega T}, \tau\right)\mathbf{a}\left(e^{j\omega T}\right)$$

$$\mathbf{d}[n] = \mathbf{N}(\tau)\mathbf{b}[n]$$

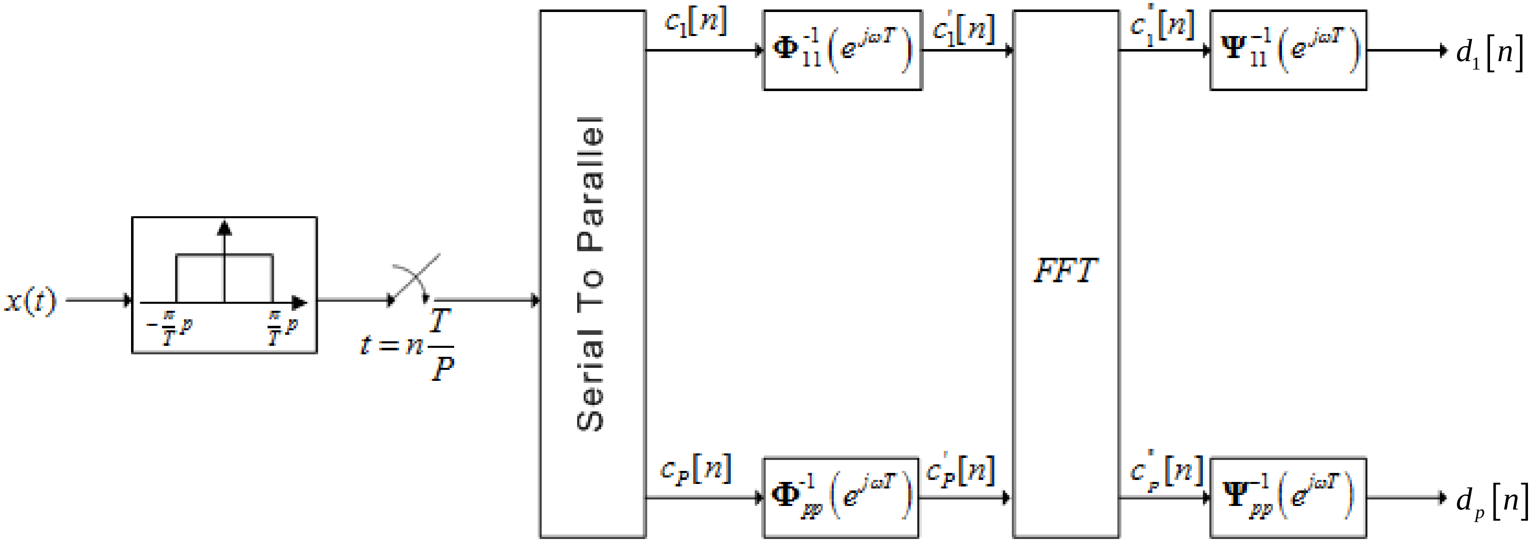
$$\mathbf{N}_{mk}(\tau) = e^{-j\frac{2\pi}{T}(m-1)t_k}$$

Vandermonde Matrix

- Set of linear measurements
  - Fits the data model of direction of arrival and spectral estimation frameworks
  - Delays can be recovered using subspace methods, such as ESPRIT and MUSIC
  - For unique solution: $p \geq 2K$
- Gain coefficients recovered by:

$$\mathbf{a}\left(e^{j\omega T}\right) = \mathbf{D}^{-1}\left(e^{j\omega T}, \tau\right)\mathbf{N}^{\dagger}(\tau)\mathbf{d}\left(e^{j\omega T}\right)$$

Practical Implementation



$$\mathbf{W}^{-1}\left(e^{j\omega T}\right) = \mathbf{\Psi}^{-1}\left(e^{j\omega T}\right)\mathbf{F}\mathbf{\Phi}^{-1}\left(e^{j\omega T}\right)$$

- $\mathbf{F}$ is the DFT matrix

$$\Phi_{ll}\left(e^{j\omega T}\right) = \sqrt{p}\left(-1\right)^{l-1} e^{j\omega(l-1)T/p}$$

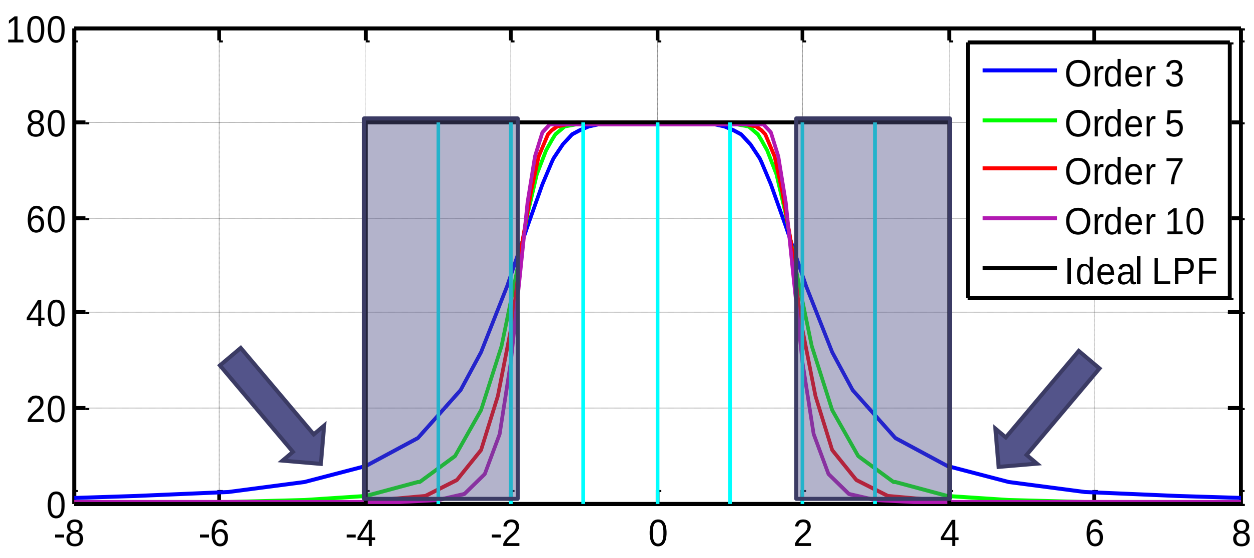
$$\Psi_{mm}\left(e^{j\omega T}\right) = \frac{1}{T} S^{*}\left(\omega + \frac{2\pi}{T}(m - p/2 - 1)\right) G\left(\omega + \frac{2\pi}{T}(m - p/2 - 1)\right)$$

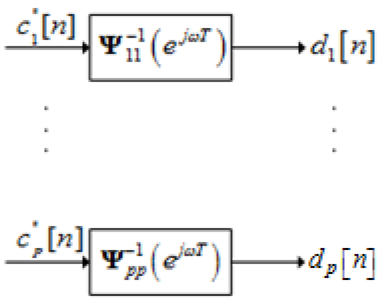
- Only one LPF and one sampling channel are used

Practical Filter - Butterworth

- We will consider this filter as band-limited from $2\omega_0$

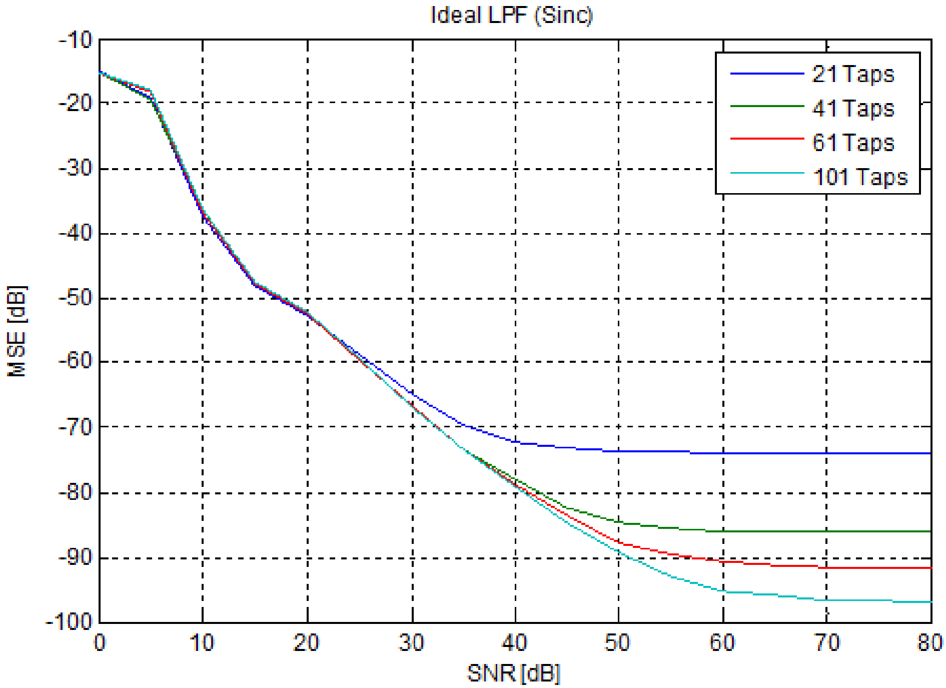
Empirically chosen

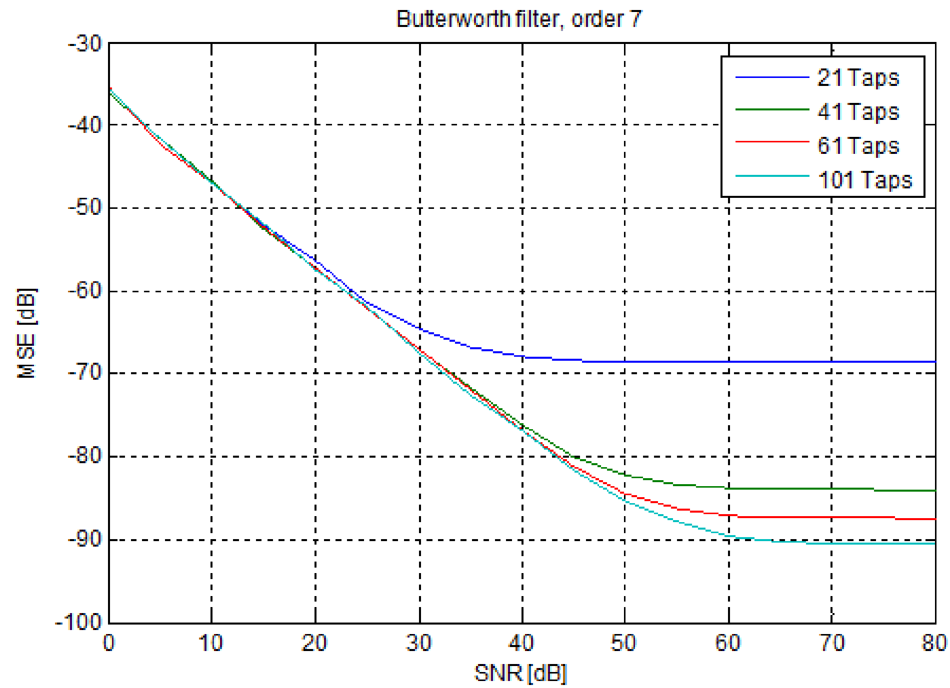




- Problems:
  - Aliasing
  - Noise enhancement at the "edges" (due to Ψ^{-1} correction)
- We will use only the central channels

Delays Estimation Results





- Performance was checked as a function of the digital filters length (should be infinite theoretically) and SNR
- Good results even when using Butterworth filter

Conclusions

- A new time delay estimation method based on sub-Nyquist sampling, was investigated
- We showed that it can be implemented using practical analog filters
- The achieved sampling rate is much lower than the traditional Nyquist rate

Reference

K. Gedalyahu and Y. C. Eldar, "Time Delay Estimation from Low Rate Samples: A Union of Subspaces Approach", *to appear in IEEE Trans. Signal Processing*.