Department of Electrical Engineering
Technion – Israel Institute of Technology

Non-Local Means Denoising Using a Content-Based Search Region and Dissimilarity Kernel

Hila Berkovich, David Malah, and Meir Barzohar

ISPA 2013

8th Int'l Symposium on Image and Signal Processing and Analysis

04-Sep-13

Outline

- Introduction
  - Image Denoising
  - Standard Non-Local Means (NLM)
- Proposed NLM Modifications
- Experimental Results
- Conclusions & Future Work

Introduction: Image Denoising

- Image denoising is used to find the best estimate of the original image given its noisy version.
- Common noise model:

$$\boxed{Y = X + N,\ \ N \sim \mathcal{N}\left(0, \sigma_n^2\right)}$$

$$Y = \text{noisy image}$$

$$X = \text{original image (unknown)}$$

$$N = \text{additive white noise}$$

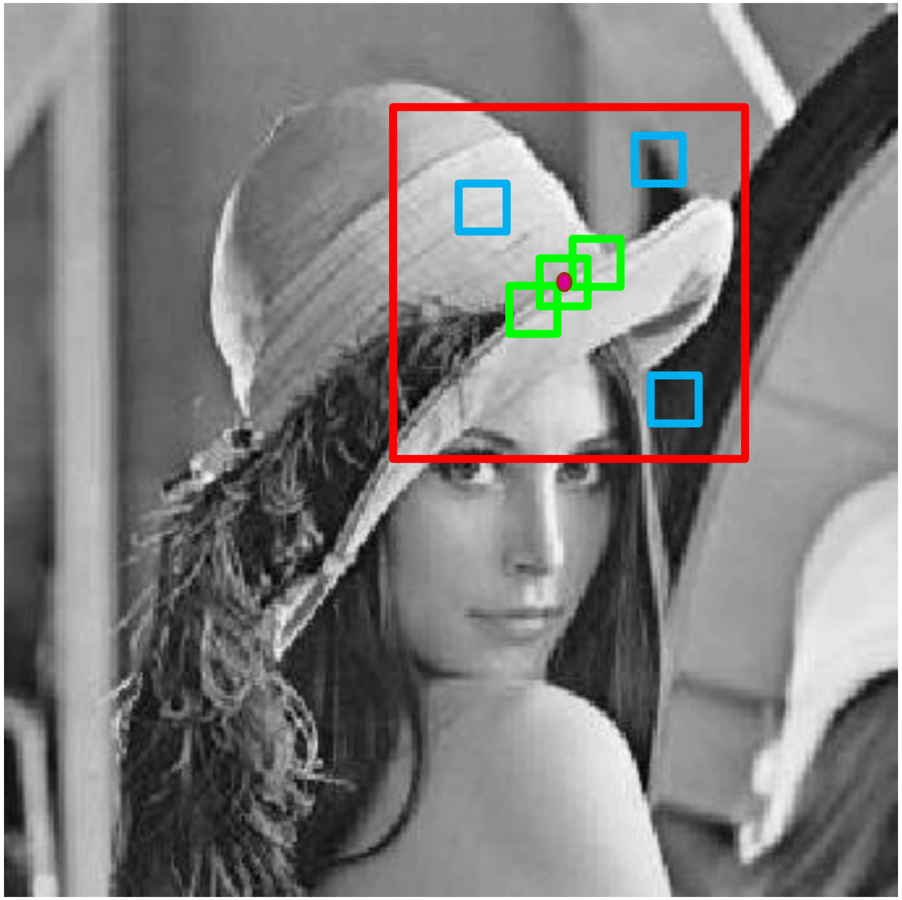
It is assumed that X and N are independent

- Patch-based denoising methods have drawn much attention.

Standard Non-Local Means (NLM)

- Introduced by Buades et. al (2005).
- Exploits image redundancy.
- Pixel restoration: Weighted average of all gray values within the defined search region S_i .

$$\hat{X}_i = \sum_{j \in S_i} w_{i,j} Y_i$$



Weights Definition

- The weights are based on similarity between pixel neighborhoods

$$w_{i,k} = \frac{1}{W_i} \exp\left(-\frac{d_p(i,k)}{h^2} \right), \; k \in S_i, \; i \text{ is the POI}$$

$$d_p(i,k) = \frac{1}{p^2} \left\| Y(A_i) - Y(A_k) \right\|_2^2$$

$d_p(i,k)$ = dissimilarity measure between neighboorhoods of pixels i and k

S_i = rectangular search region of size $M \times M$

A_i = similarity patch of size $p \times p$

h = weight smoothing parameter

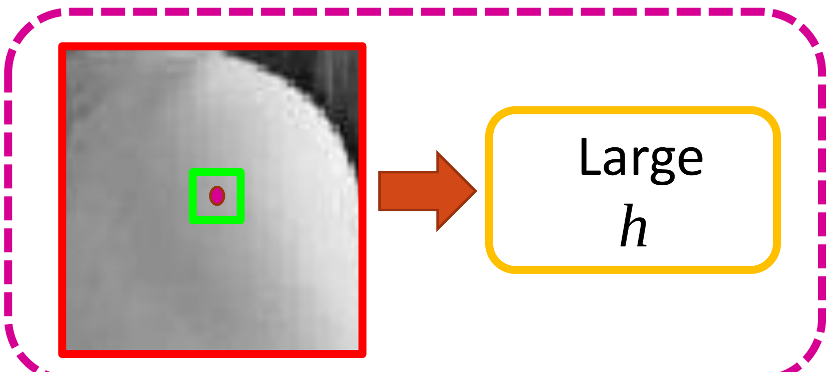
W_i = normalization factor $\left(\sum_{k \in S_i} w_{i,k} \right)$

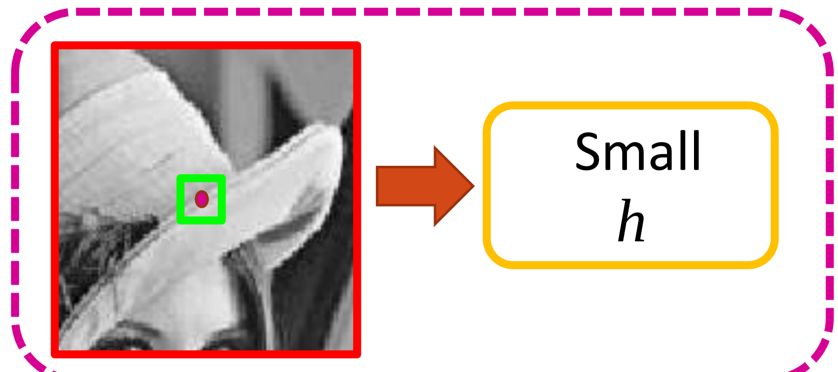
The Parameter *h*

- The NLM algorithm is sensitive to the selection of the parameter *h*

$$w_{i,j} = \frac{1}{W_i} e^{-\frac{d_p(i,j)}{h^2}}, \quad j \in S_i$$

- It is usually set to be proportional to σ_n .
- In addition, simulations suggest that *h* should match local structure:



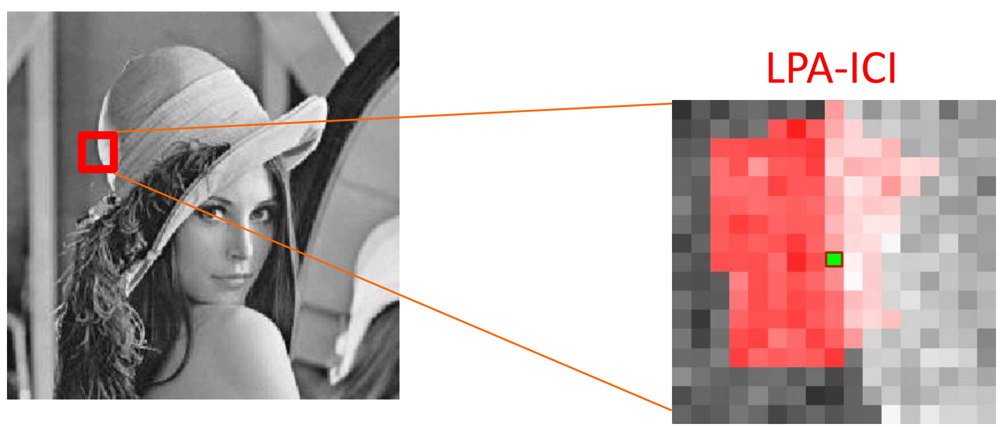


- There are NLM modifications that suggest to use an adaptive *h*, matched to local structure (e.g., Duval et al. 2010, Dinesh et al. 2009)

High computational complexity

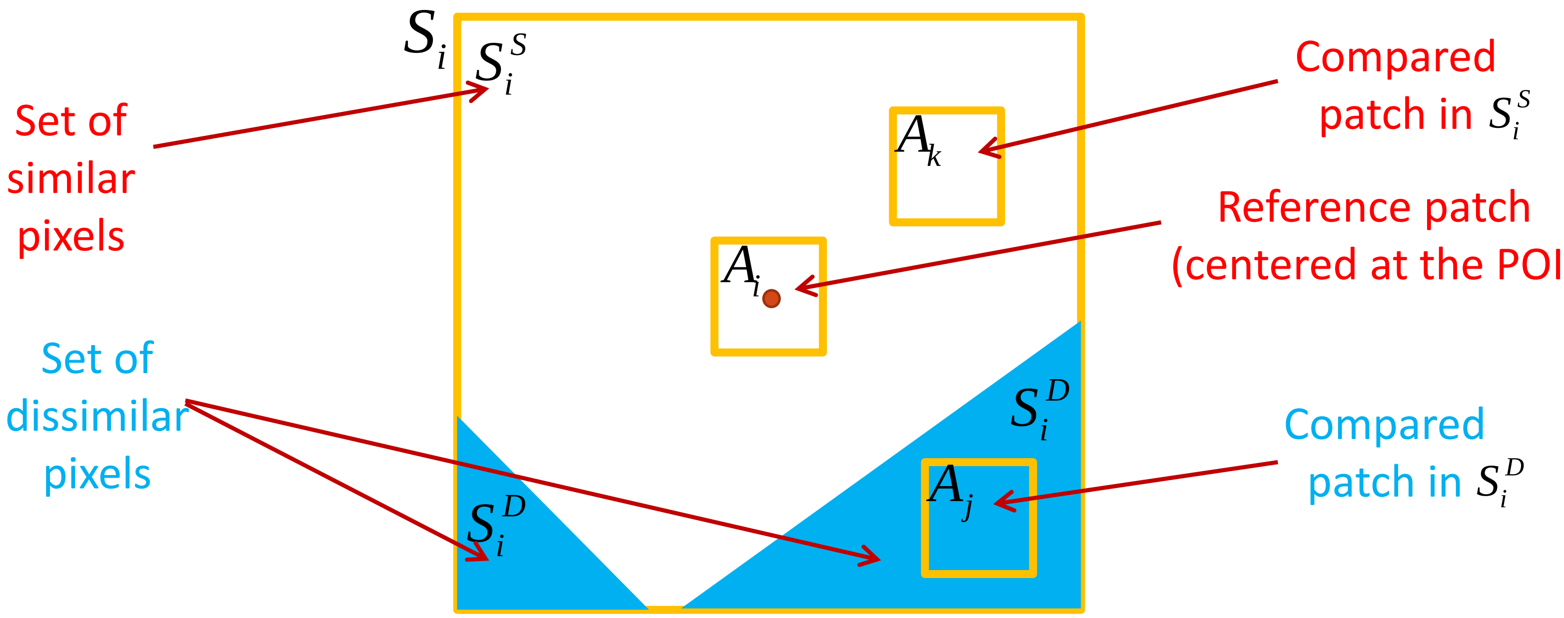
Alternative for Using a Local *h* – Adaptive Search Region

- Method: use an anisotropic **adaptive** region, which includes only pixels with similar neighborhoods to that of the POI.
- Prior art:
  - Gradient-based classification (Mahmoudi et al. 2005) – sensitive to noise
  - Similarity patch correlation (Dinesh et al. 2009) – a threshold is required
  - **L**ocal **P**olynomial **A**pproximation combined with the **I**ntersection of **C**onfidence **I**ntervals (LPA-ICI) (Sun et al. 2009) – complex and enforces contiguity of search region



Creates wide edge → causes over-smoothing

Proposed Modification I: Adaptive Model-Based Search Region



Assumptions:

$$\forall k \in S_i^S \setminus \{i\}: \; X(A_i) = X(A_k) \rightarrow Y(A_i) - Y(A_k) = N(A_i) - N(A_k)$$

$$\forall j \in S_i^D: \; X(A_i) = C_j + X(A_j) \rightarrow Y(A_i) - Y(A_j) = C_j + N(A_i) - N(A_j)$$

Distribution of Dissimilarity Measure

- A Compared patch included in S_i^S:

$$\forall k \in S_i^S \setminus \{i\}: \quad \frac{d_p(i,k)}{2\sigma_n^2} = \frac{1}{p^2}\frac{\left\| Y(A_i) - Y(A_k) \right\|_2^2}{2\sigma_n^2} = \frac{1}{p^2}\sum_{\substack{m \in A_i \\ l \in A_k}} \left(\frac{N_m - N_l}{\sqrt{2}\sigma_n} \right)^2 \sim \chi_{p^2}^2$$

Chi-Square

$$\sim \mathcal{N}(0,1)$$

$$\Rightarrow \boxed{\mathrm{E}\left[\frac{d_p(i,k)}{2\sigma_n^2}\right] = 1, \quad Var\left[\frac{d_p(i,k)}{2\sigma_n^2}\right] = \frac{2}{p^2}}$$

- A Compared patch included in S_i^D:

Non-Central Chi-Square

$$\forall j \in S_i^D: \quad \frac{d_p(i,j)}{2\sigma_n^2} = \frac{1}{p^2}\frac{\left\| Y(A_i) - Y(A_j) \right\|_2^2}{2\sigma_n^2} = \frac{1}{p^2}\sum_{\substack{m \in A_i \\ l \in A_k}} \left(\frac{C_j + N_m - N_l}{\sqrt{2}\sigma_n} \right)^2 \sim \chi_{p^2}^2(\lambda_j)$$

$$\lambda_j = f(C_j)$$

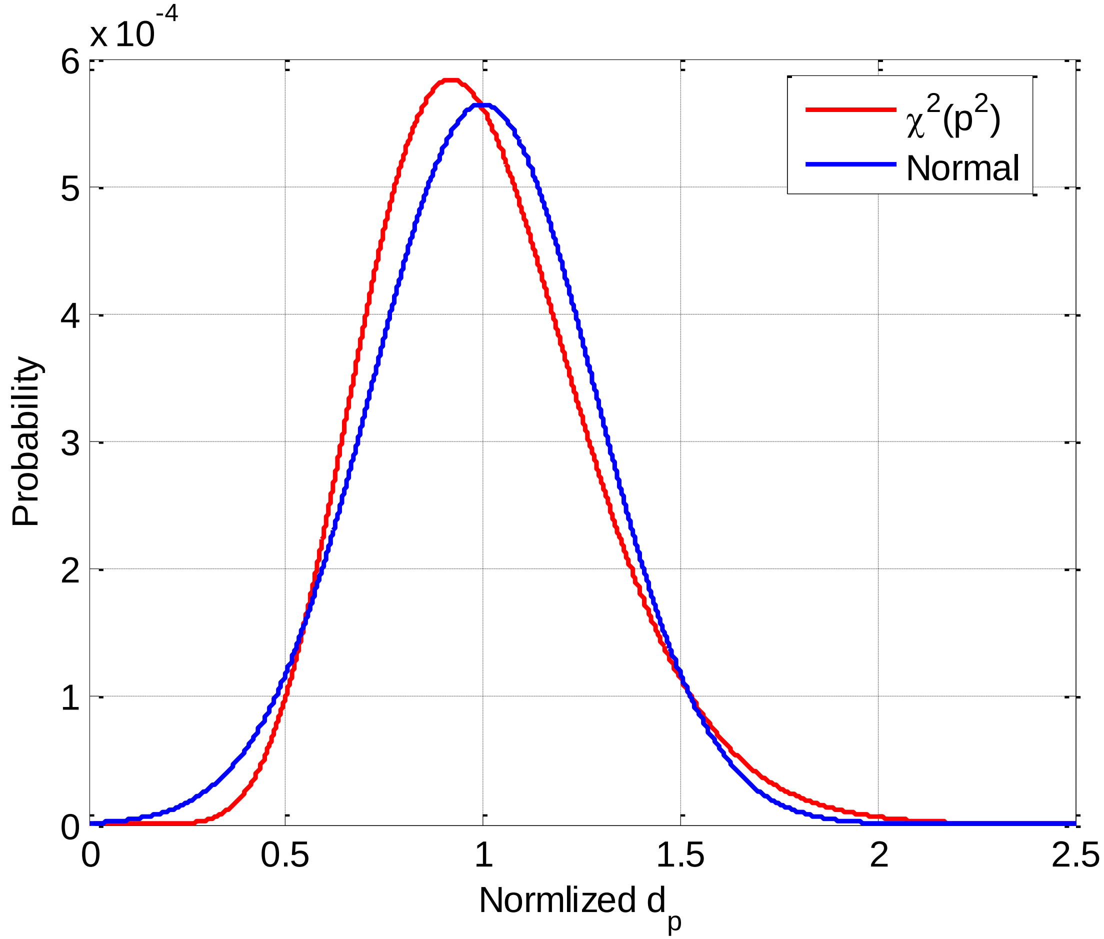
$$\Rightarrow \boxed{\mathrm{E}\left[\frac{d_p(i,j)}{2\sigma_n^2}\right] = 1 + \frac{\lambda_j}{p^2}, \quad Var\left[\frac{d_p(i,j)}{2\sigma_n^2}\right] = \frac{2}{p^2} + \frac{4\lambda_j}{p^4}}$$

$$\sim \mathcal{N}\left(\frac{C_j}{\sqrt{2}\sigma_n}, 1\right)$$

Distribution Approximation

- For $p^2 \gg 1$, the Chi-Square distribution converges to a Normal distribution.

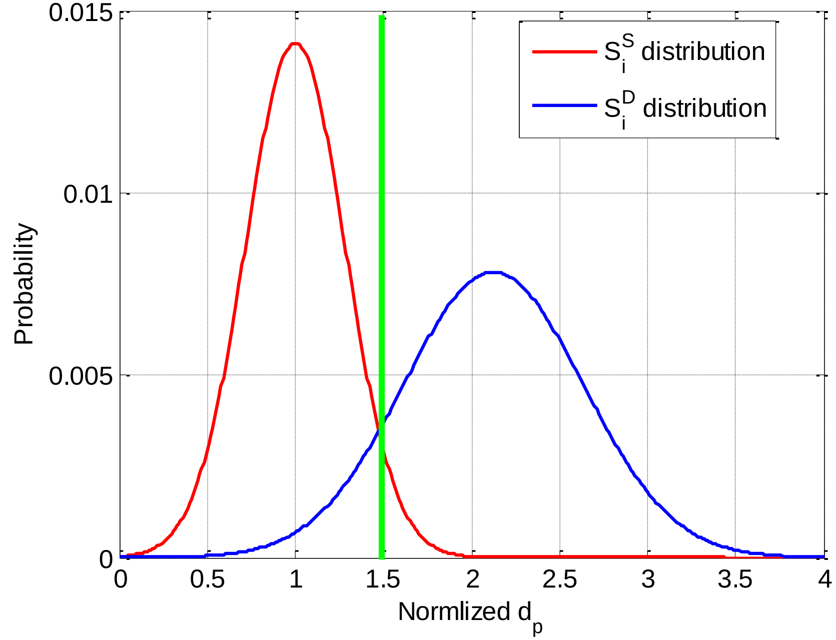
For $p^2 = 25$



Difference Between Distributions

$$\forall k \in S_i^S \setminus \{i\}: \quad \frac{d_p(i,k)}{2\sigma_n^2} \sim \mathcal{N}\left(1, \frac{2}{p^2}\right)$$

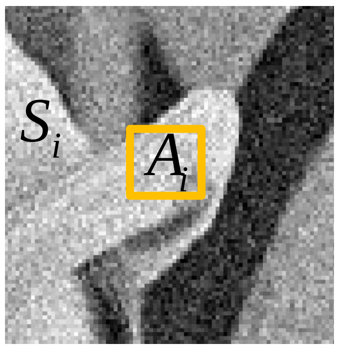
$$\forall j \in S_i^D: \quad \frac{d_p(i,j)}{2\sigma_n^2} \sim \mathcal{N}\left(1+\frac{\lambda_j}{p^2}, \frac{2}{p^2}+\frac{4\lambda_j}{p^4}\right)$$

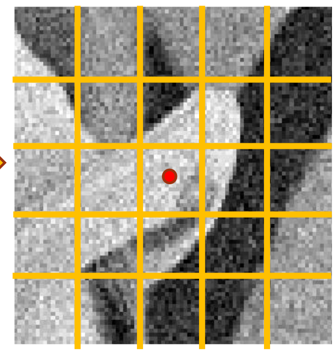


- The difference between the distributions of the two sets can serve as a classification measure.
- Since λ_j is unknown, we use a one-side hypothesis based on the dissimilarity variance:

Pixels included in S_i^S are characterized by a variance $\leq 2/p^2$

Classification Via Accumulated Variance





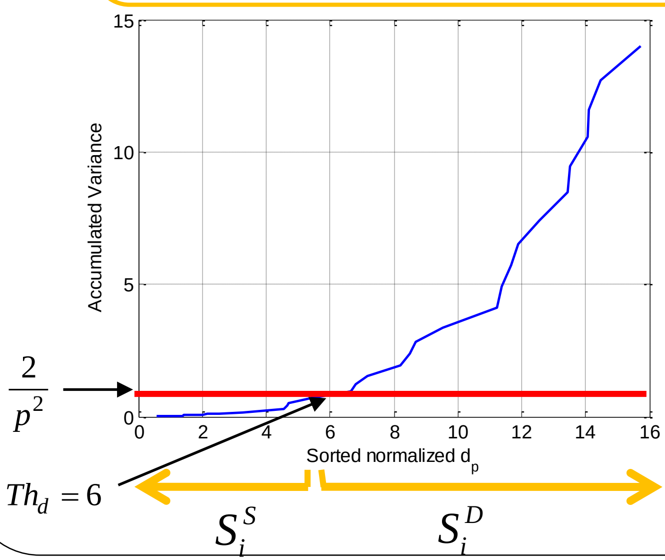
$$\forall k \in S_i : \frac{d_p(i,k)}{2\sigma_n^2}$$

Sort $\left\{ \frac{d_p(i,k)}{2\sigma_n^2} \right\}_{k \in S_i}$ in an ascending order

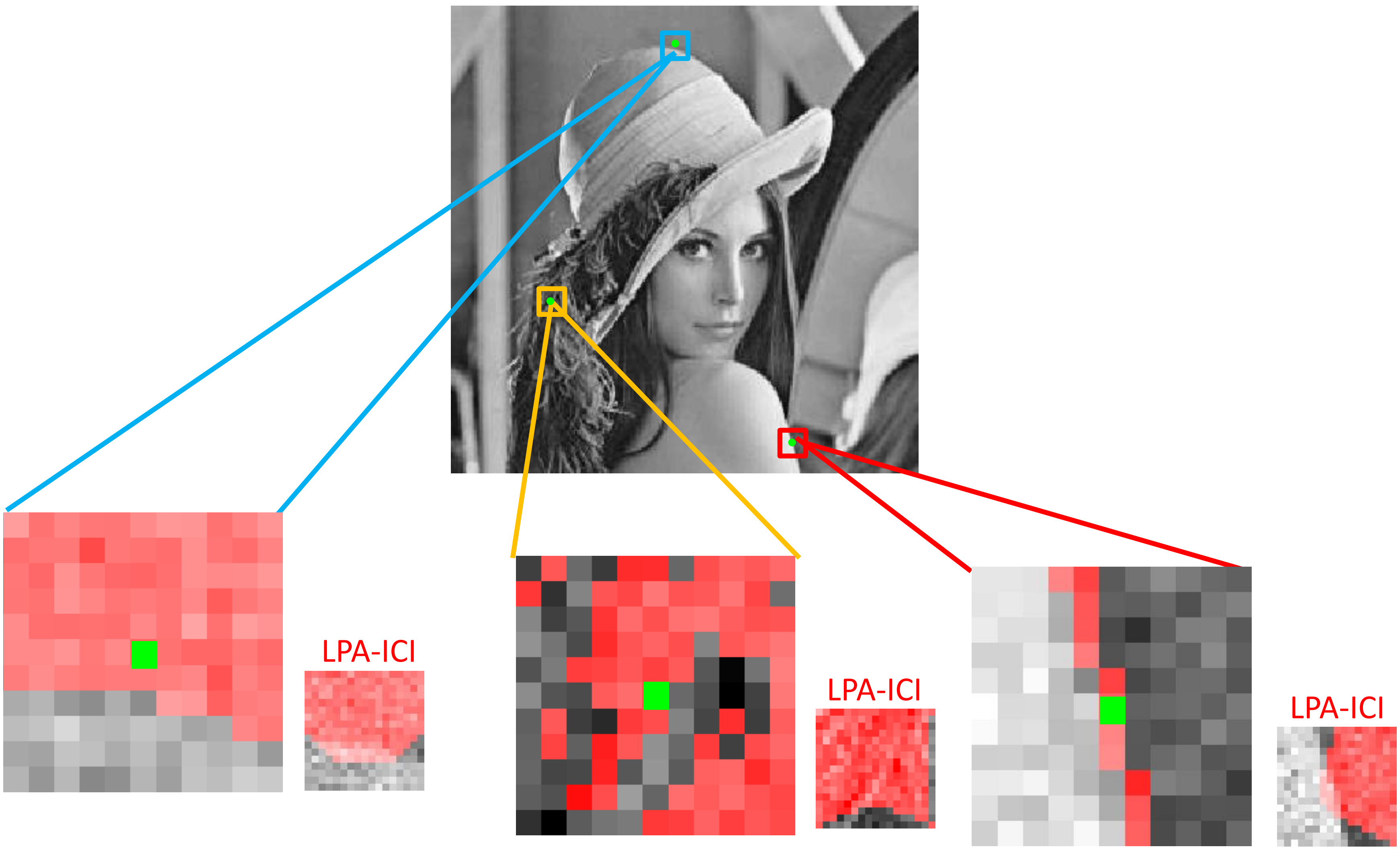
Compute **Accumulated Variance** by starting with the first 2 elements and adding one element at a time

Stop accumulation once

$$Var\left\{ \frac{d_p(i,\mathrm{k})}{2\sigma_n^2} \right\}_{k \in S_i} > \frac{2}{p^2}$$



Examples of Adaptive Search Region of Different Local Structures

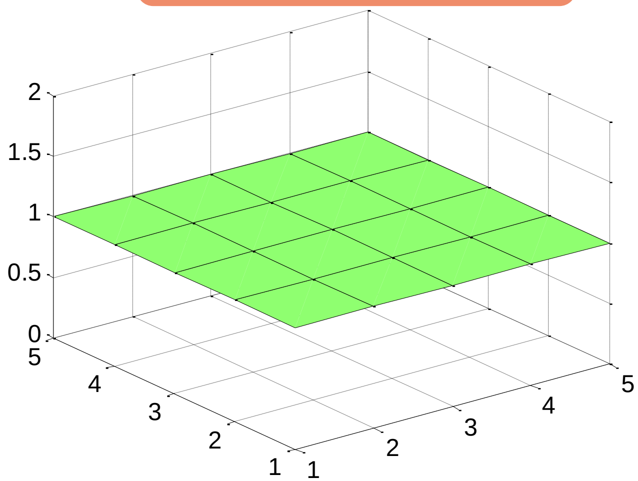


NLM with Patch–Kernel

- 2 types of patch (dissimilarity)-kernels are used frequently in NLM denoising:

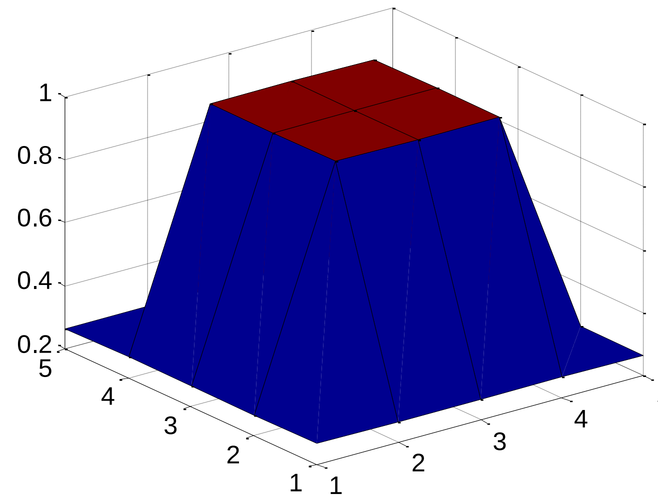
$$d_p(i,k) = \frac{1}{p^2}\left\| Y(A_i) - Y(A_k) \right\|_{2,a}^2 = \frac{1}{p^2}\sum_{\substack{m\in A_i \\ l\in A_k}} \alpha_m \left(N_m - N_l\right)^2$$

Uniform patch-kernel



Smooth regions

Box patch-kernel

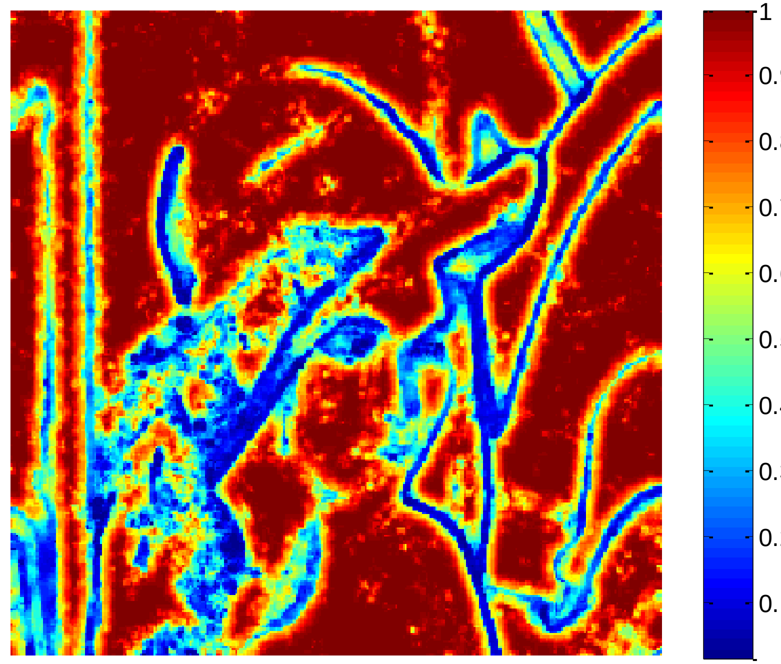


Textured regions / Edges

Proposed Modification II: Patch–Kernel Type Adaptation

- The Adaptive Model-Based Search Region output provides an S_i^S set per pixel.

Normalized Cardinality map $\frac{\left|S_i^S\right|}{M^2}$



Cluster Cardinality Map Data

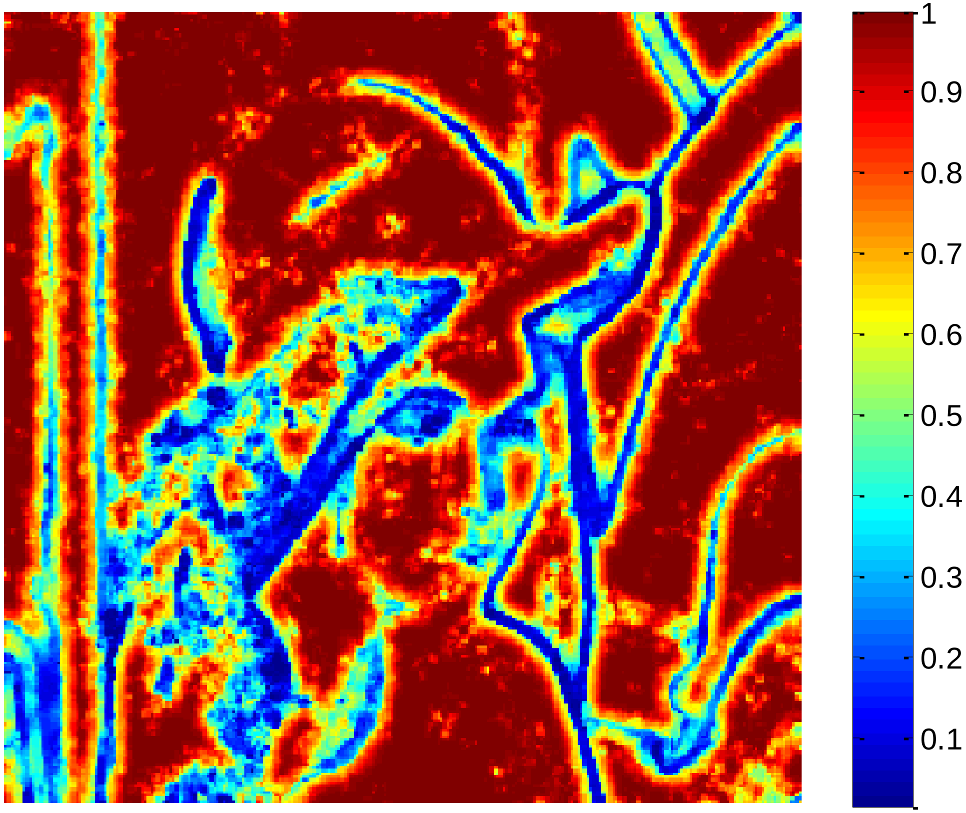
- Classify the data of the normalized cardinality map using K-Means with K=2.
- The classification results in 2 centroids:

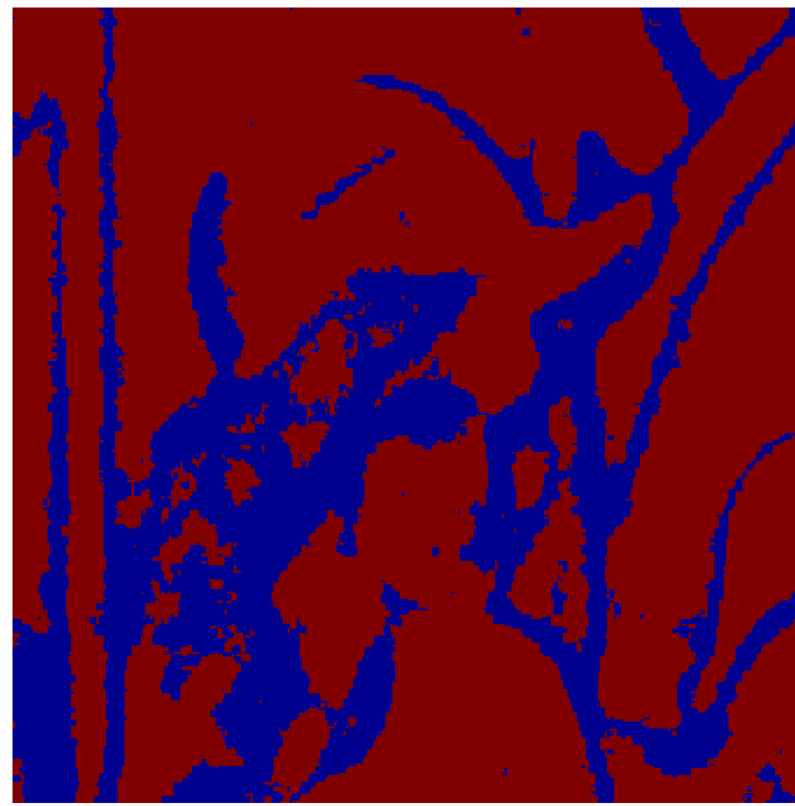
Large centroid value	Small centroid value
Weights are computed based on **Uniform** patch-kernel	Weights are computed based on **Box** patch-kernel

Patch–Kernel Type Adaptation (Cont'd)

- Cardinality map clustered data $\frac{|S_i^S|}{M^2}$

  * For $\sigma_n = 20$





Experimental Results

- Uniform NLM vs. Adaptive NLM

	Uniform	Adaptive
PSNR [dB]	24.78	25.62
SSIM	0.689	0.75

* For $\sigma_n = 20$

$p = 5$

$M = 11$

Uniform NLM

Experimental Results (Cont'd)

- Uniform NLM vs. Adaptive NLM

	Uniform	Adaptive
PSNR [dB]	24.78	25.62
SSIM	0.689	0.75

* For $\sigma_n = 20$

$p = 5$

$M = 11$

Adaptive NLM

Experimental Results (Cont'd)

- Box NLM vs. Adaptive NLM

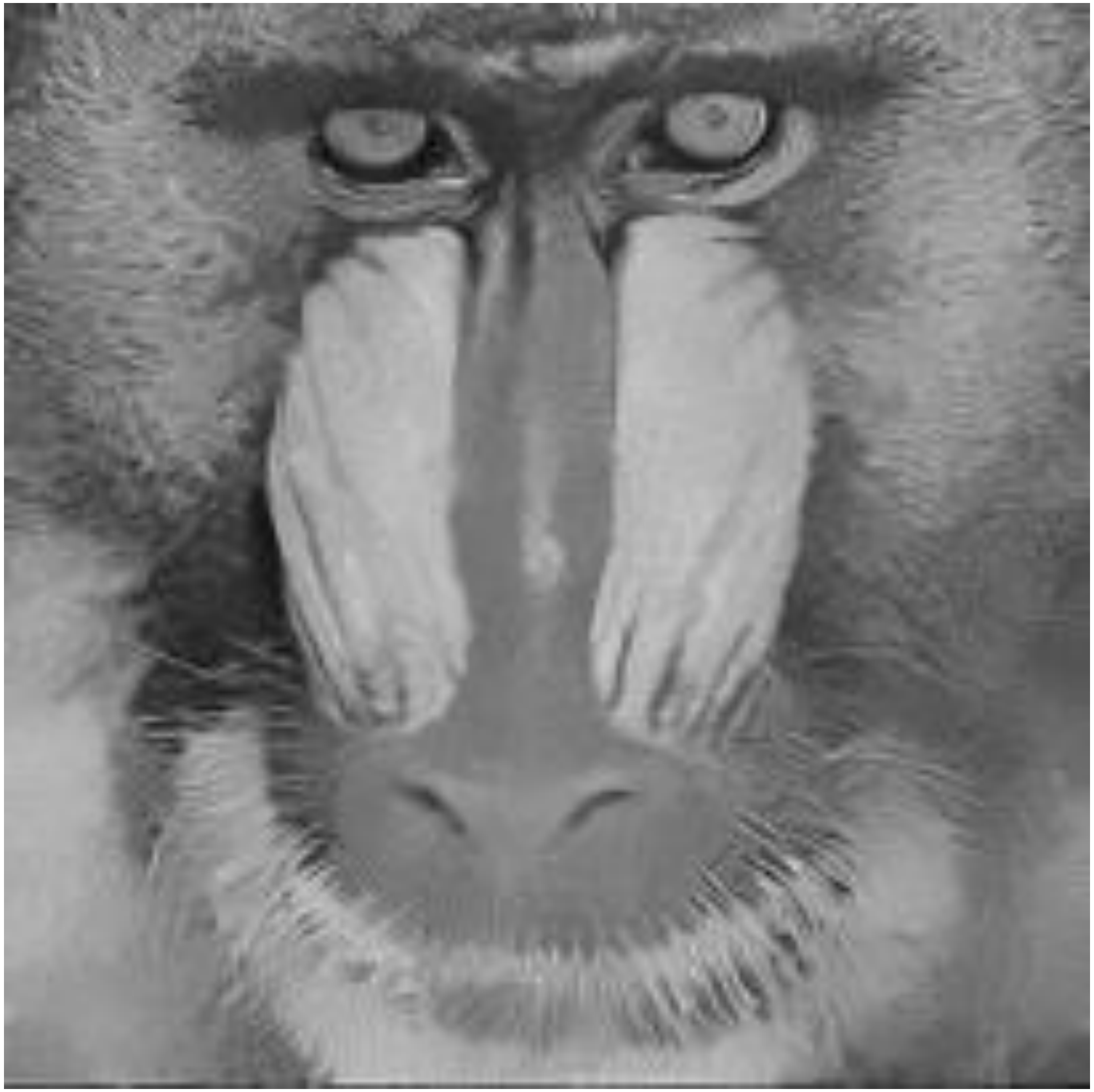
	Box	Adaptive
PSNR [dB]	25.54	25.62
SSIM	0.74	0.75

* For $\sigma_n = 20$

$p = 5$

$M = 11$

Box NLM



Experimental Results (Cont'd)

- Box NLM vs. Adaptive NLM

	Box	Adaptive
PSNR [dB]	25.54	25.62
SSIM	0.74	0.75

* For $\sigma_n = 20$

Adaptive NLM

Experimental Results (Cont'd)

Image	Noise Level	NLM with Uniform Kernel PSNR [dB] /SSIM	NLM with Box Kernel PSNR [dB] /SSIM	Proposed Adaptive Approach PSNR [dB] /SSIM
Lena	20	30.11/0.87	30.27/0.86	**30.48/0.88**
Baboon	20	24.78/0.69	25.54/0.74	**25.62/0.75**
Barbara	20	29.11/0.87	29.19/0.87	**29.33/0.88**
Lena	30	28.03/0.81	28.03/0.78	**28.32/0.82**
Pepper	30	28.03/0.83	28.06/0.81	**28.39/0.84**

Conclusion

- Two modifications of the NLM algorithm were introduced:
  - **Model-based** adaptive search region
    - Parameter-free
    - not restricted to be contiguous
  - **Content-based** patch-kernel type
    - matched to local structure → smooth regions are less granular while texture and edges are preserved.

Future Work

- Consider correlation between dissimilarity values due to:
  - Overlap between similarity patches.
  - Patches in the same search region are compared to the same reference patch.
- Apply the suggested modifications on images characterized by Poisson noise.

In Progress

Non-Local Means Denoising Using a Content-Based Search Region and Dissimilarity Kernel

ISPA 2013

8th Int'l Symposium on Image and Signal Processing and Analysis